On the Analytical Approach to the *N*-Fold Bäcklund Transformation of Davey-Stewartson Equation

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Abstract

N-fold Bäcklund transformation for the Davey-Stewartson equation is constructed by using the analytic structure of the Lax eigenfunction in the complex eigenvalue plane. Explicit formulae can be obtained for a specified value of *N*. Lastly it is shown how generalized soliton solutions are generated from the trivial ones.

Introduction: Inverse scattering transform holds a central place in the analysis of nonlinear integrable system in either (1+1)- or (2+1)-dimensions [1]. On the other hand it has been found that explicit soliton solutions can also be obtained by the use of Bäcklund transformations in a much easier way [2]. These Bäcklund transformations are also useful in proving the superposition formulae for these solutions. There have been many attempts to construct explicit *N*-soliton solutions for nonlinear integrable system either by Bäcklund transformations or the inverse scattering method. A separate and elegant approach was developed by Zakharov *et al* [3] which relied on the pole structure of the Lax eigenfunction and use of projection operators. In this letter we have used an approach similar to that of Zakharov *et al* but have generated a formulae for the *N*-fold BT of the nonlinear field variables occurring in the (2+1)-dimensional Davey-Stewartson equation [4]. Our approach is very similar to that of gauge transformation repeatedly applied to any particular seed solution. Lastly we demonstrate how non-trivial solutions are generated by starting with known trivial ones.

Formulation: The Davey-Stewartson equation under consideration can be written as

$$ir_t + r_{xx} - r_{yy} + r(A_2 - A_1) = 0,$$
$$iq_t + q_{yy} - q_{xx} + q(A_1 - A_2) = 0,$$
$$A_{1x} = -\frac{1}{2}(q_y r + r_y q), \qquad A_{2y} = -\frac{1}{2}(q_x r + r_x q). \tag{1}$$

Equation (1) is known to be a result of the consistency of the operators T_1 and T_2 written as $[T_1, T_2]\Psi = 0$, where

$$T_1\Psi = \left\{ 2 \begin{pmatrix} \partial_x & 0 \\ 0 & \partial_y \end{pmatrix} + \begin{pmatrix} 0 & q \\ r & 0 \end{pmatrix} \right\} \Psi = 0,$$

$$T_2\Psi = \frac{1}{2} \left\{ (i\partial_t + \partial_x^2 + \partial_y^2) + \begin{pmatrix} 0 & q_x \\ r_y & 0 \end{pmatrix} + A \right\} \Psi = -\frac{K^2}{2}\Psi,$$

and $A = \begin{pmatrix} A_1 & 0 \\ 0 & A_2 \end{pmatrix}$, $K = \text{const.}$

To proceed further we set

$$A_1 = f_{1y}, \qquad A_2 = f_{2x}$$

and

$$\Psi = \Phi \exp\left\{ i\left(\alpha + \lambda^{-2}\right) x - i\left(\beta - \lambda^{-2}\right) y \right\}.$$

Whence the Lax pair becomes,

$$M\Phi = U\Phi, \qquad \Phi_t = V\Phi \tag{2}$$

with

$$M = \begin{pmatrix} \partial_x & 0 \\ 0 & \partial_y \end{pmatrix}, \qquad U = \begin{pmatrix} -i\left(\alpha + \lambda^{-2}\right) & -q/2 \\ -r/2 & i\left(\beta - \lambda^{-2}\right) \end{pmatrix} \tag{3}$$

and

$$V = \begin{pmatrix} i\Lambda + Q(\partial_x, \partial_y) + if_{1y} & iq_x \\ ir_y & i\Lambda + Q(\partial_x, \partial_y) + if_{2y} \end{pmatrix}. \tag{4}$$

With

$$\Lambda = K^2 - \left(\alpha + \lambda^{-2}\right)^2 - \left(\beta - \lambda^{-2}\right)^2,$$

$$Q(\partial_x, \partial_y) = i\left(\partial_x^2 + \partial_y^2\right) - 2\left\{ \left(\alpha + \lambda^{-2}\right)\partial_x - \left(\beta - \lambda^{-2}\right)\partial_y \right\},$$

we can construct particular Jost solutions, corresponding to $q = q_0 = \text{const}$, $r = r_0 = \text{const}$, $A_1 = A_{10} = f_{1y}^0$ and $A_2 = A_{20} = f_{2x}^0$ with $f_{1y}^0 = f_{2x}^0 = \text{const}$. This particular eigenvector Φ_0 turns out to be

$$\hat{\Phi}_0 = \begin{pmatrix} \exp(\theta_1 x + \chi_1 y + \xi_1 t) & \exp(\theta_2 x + \chi_2 y + \xi_2 t) \\ m_0 \exp(\theta_1 x + \chi_1 y + \xi_1 t) & n_0 \exp(\theta_2 x + \chi_2 y + \xi_2 t) \end{pmatrix}$$

with

$$\theta_1 = -i\left(\alpha + \lambda^{-2}\right) + am_0, \qquad \theta_2 = -i\left(\alpha + \lambda^{-2}\right) + an_0,$$
$$\chi_1 = b/m_0 + i\left(\beta - \lambda^{-2}\right), \qquad \chi_2 = b/n_0 + i\left(\beta - \lambda^{-2}\right),$$

ξ_1, ξ_2 are arbitrary complex constants, m_0, n_0 are arbitrary constants and $m_0 \neq n_0$.

Note that $\det \Phi_0 \neq 0$ so that Φ_0^{-1} exists.

Now suppose that Φ_{n-1} denotes the Lax eigenfunction, corresponding to the $(n-1)$ soliton solution, and $B_n(x,y,t)$ be the transformation which yields the Φ_n (solution corresponding to the n soliton case when applied to Φ_{n-1}), that is

$$\Phi_n(x,y,t,\lambda) = B_n(x,y,t,\lambda)\Phi_{n-1}(x,y,t,\lambda).$$

Using the above Lax equations (2) and (4), we can at once deduce the equations satisfied by B_n

$$MB_n = U_nB_n - B_nU_{n-1}, \qquad \partial_t B_n = V_nB_n - B_nV_{n-1}, \tag{5}$$

where U_n, V_n denote the Lax matrices.

Corresponding to the n-soliton solution: Note that U, V are even functions of λ, so that we can assume that

$$B_n(-\lambda) = B_n(\lambda).$$

We now assume B_n to have simple pole structure in the complex λ-plane, so that

$$B_n(x,y,t,\lambda) = Q_n + \frac{2\lambda_n}{\lambda^2 - \lambda_n^2}P_n. \tag{6}$$

We also assume that

$$B_n^{-1}(x,y,t,\lambda) = Q_n' + \frac{2\lambda_n'}{\lambda^2 - \lambda_n^2}P_n', \tag{7}$$

where P_n, Q_n, P_n', Q_n' are matrix functions of (x,y,t). The condition $B_nB_n^{-1} = B_n^{-1}B_n = I$ leads to

$$B_n\left(\lambda_n'\right)P_n' = 0, \qquad P_nB_n^{-1}(\lambda_n) = 0,$$
$$B_n^{-1}(\lambda_n)P_n = 0, \qquad P_n'B_n\left(\lambda_n'\right) = 0.$$

Calculation of the matrices Q_n, P_n, Q_n', P_n'**:** Let us now go back to equation (5) and use the expression (6) and (7). Rewriting U_n as

$$U_n = -i\lambda^{-2}I + U_n'$$

and using the form of B_n given in (6) we get:

$$MQ_n + \frac{2\lambda_n}{\lambda^2 - \lambda_n^2}(MP_n) = \left(-i\lambda^{-2}I + U_n'\right)\left(Q_n + \frac{2\lambda_n}{\lambda^2 - \lambda_n^2}P_n\right)$$
$$- \left(Q_n + \frac{2\lambda_n}{\lambda^2 - \lambda_n^2}P_n\right)\left(-i\lambda^{-2}I + U_{n-1}'\right)$$

which yields equations satisfied by P_n and Q_n:

$$M(P_n\Phi_{n-1}(\lambda_n)) = U_n(\lambda_n)(P_n\Phi_{n-1}(\lambda_n)),$$
$$M(Q_n\Phi_{n-1}(\lambda_n)) = U_n(\lambda_n)(Q_n\Phi_{n-1}(\lambda_n)).$$

Using the same form of V_n as give in equation (3) in the time part, we get the following equations:

$$\partial_x\left(Q_n - \frac{2}{\lambda_n}P_n\right) = -\partial_y\left(Q_n - \frac{2}{\lambda_n}P_n\right),$$
$$\partial_t(P_n\Phi_{n-1}(\lambda_n)) = V_n(\lambda_n)(P_n\Phi_{n-1}(\lambda_n)),$$
$$Q_{nt} = OP_2Q_n + 2i(Q_{nx}\partial_x + Q_{ny}\partial_y) + D_nQ_n - Q_nD_{n-1},$$

where operator $OP_2 \equiv i\left(\partial_x^2 + \partial_y^2\right) - 2(\alpha\partial_x - \beta\partial_y)$, $\alpha, \beta = $ const., and $D_n = \begin{pmatrix} if_{1y}^n & iq_{nx} \\ ir_{ny} & if_{2x}^n \end{pmatrix}$.

As per the ansatz of Zakharov we search for P_n in the form,

$$P_n = \begin{pmatrix} \gamma_{n1} \\ \gamma_{n2} \end{pmatrix} (\delta_{n1}, \delta_{n2}).$$

It is interesting to observe that

$$(\delta_{n1}, \delta_{n2}) = (a_{n1}, a_{a2})\Phi_{n-1}^{-1}(n),$$

where a_{n1}, a_{n2} are practically two constants.

Similarly for P_n' and Q_n', we set

$$P_n' = \begin{pmatrix} \gamma_{n1}' \\ \gamma_{n2}' \end{pmatrix} (\delta_{n1}', \delta_{n2}') \quad \text{and} \quad Q_n' = \begin{pmatrix} \alpha_n' & \alpha_n'' \\ \beta_n' & \beta_n'' \end{pmatrix}.$$

Whence we get

$$B_l(\lambda_l') = \begin{pmatrix} F_l^{11} + \sigma_l\gamma_{l1}\delta_{l1} & F_l^{12} + \sigma_l\gamma_{l2}\delta_{l2} \\ F_l^{21} + \sigma_l\gamma_{l2}\delta_{l1} & F_l^{22} + \sigma_l\gamma_{l2}\delta_{l2} \end{pmatrix},$$

where

$$F_l^{11} = f_l^{11}(t)\exp\{im_{1l}(x-y)\}, \qquad F_l^{12} = f_l^{12}(t)\exp\{im_{1l}'(x-y)\},$$
$$F_l^{21} = f_l^{21}(t)\exp\{im_{2l}(x-y)\}, \qquad F_l^{22} = f_l^{22}(t)\exp\{im_{2l}'(x-y)\},$$

and

$$\sigma_l = \frac{2}{\lambda_l} - \frac{2\lambda_l}{\lambda_l^2 - \lambda_l'^2},$$

$$\delta_{l1} = -\frac{\delta_{l1}'F_l^{11} + \delta_{l2}'F_l^{21}}{\sigma_l(\delta_{l1}'\gamma_{l1} + \delta_{l2}'\gamma_{l2})}, \qquad \delta_{l2} = -\frac{\delta_{l1}'F_l^{12} + \delta_{l2}'F_l^{22}}{\sigma_l(\delta_{l1}'\gamma_{l1} + \delta_{l2}'\gamma_{l2})},$$

$$\gamma_{l1} = -\frac{\gamma_{l1}'F_l^{11} + \gamma_{l2}'F_l^{12}}{\sigma_l(\gamma_{l1}'\delta_{l1} + \gamma_{l2}'\delta_{l2})}, \qquad \gamma_{l2} = -\frac{\gamma_{l1}'F_l^{21} + \gamma_{l2}'F_l^{22}}{\sigma_l(\gamma_{l1}'\delta_{l1} + \gamma_{l2}'\delta_{l2})}.$$

$$B_l^{-1}(\lambda_l) = \begin{bmatrix} \alpha_l' + \varepsilon_l'\gamma_{l1}'\delta_{l1}' & \alpha_l'' + \varepsilon_l'\gamma_{l1}'\delta_{l2}' \\ \beta_l' + \varepsilon_l'\gamma_{l2}'\delta_{l1}' & \beta_l'' + \varepsilon_l'\gamma_{l2}'\delta_{l2}' \end{bmatrix}.$$

Here $\varepsilon_l' = \dfrac{2\lambda_l'}{\lambda_l^2 - \lambda_l'^2}$, along with

$$\delta_{l1}' = -\frac{\alpha_l'\delta_{l1} + \beta_l'\delta_{l2}}{\varepsilon_l'(\delta_{l1}\gamma_{l1}' + \delta_{l2}\gamma_{l2}')}, \qquad \delta_{l2}' = -\frac{\alpha_l''\delta_{l1} + \beta_l''\delta_{l2}}{\varepsilon_l'(\delta_{l1}\gamma_{l1}' + \delta_{l2}\gamma_{l2}')},$$

$$\gamma_{l1}' = -\frac{\gamma_{l1}\alpha_l' + \gamma_{l2}\alpha_l''}{\varepsilon_l'(\gamma_{l1}\delta_{l1}' + \gamma_{l2}\delta_{l2}')}, \qquad \gamma_{l2}' = -\frac{\gamma_{l1}\beta_l' + \gamma_{l2}\beta_l''}{\varepsilon_l'(\gamma_{l1}\delta_{l1}' + \gamma_{l2}\delta_{l2}')}.$$

Finally the matrix Q_l is given as

$$Q_l = \begin{pmatrix} f_l^{11}(t)\exp\{im_{1l}(x-y)\} + \dfrac{2}{\lambda_l}\gamma_{l1}\delta_{l1} & f_l^{12}(t)\exp\{im_{1l}'(x-y)\} + \dfrac{2}{\lambda_l}\gamma_{l1}\delta_{l2} \\ f_l^{21}(t)\exp\{im_{2l}(x-y)\} + \dfrac{2}{\lambda_l}\gamma_{l2}\delta_{l2} & f_l^{22}(t)\exp\{im_{2l}'(x-y)\} + \dfrac{2}{\lambda_l}\gamma_{l2}\delta_{l2} \end{pmatrix}.$$

It is also very convenient to rewrite the matrix elements of Q and P in terms of Lax eigenfunctions Φ. We collect these results below without giving the detailed derivation,

$$Q_l^{11} = \frac{R_l \Phi_{l-1}^{21}(\lambda_l) + R_l' \Phi_{l-1}^{22}(\lambda_l)}{a_l \Phi_{l-1}^{22}(\lambda_l) - b_l \Phi_{l-1}^{21}(\lambda_l)}, \qquad Q_l^{12} = \frac{M_l \Phi_{l-1}^{21}(\lambda_l) + M_l' \Phi_{l-1}^{22}(\lambda_l)}{a_l \Phi_{l-1}^{22}(\lambda_l) - b_l \Phi_{l-1}^{21}(\lambda_l)},$$

$$Q_l^{21} = \frac{L_l \Phi_{l-1}^{21}(\lambda_l) + L_l' \Phi_{l-1}^{22}(\lambda_l)}{a_l \Phi_{l-1}^{22}(\lambda_l) - b_l \Phi_{l-1}^{21}(\lambda_l)}, \qquad Q_l^{22} = \frac{N_l \Phi_{l-1}^{21}(\lambda_l) + N_l' \Phi_{l-1}^{22}(\lambda_l)}{a_l \Phi_{l-1}^{22}(\lambda_l) - b_l \Phi_{l-1}^{21}(\lambda_l)},$$

where

$$R_l = -\frac{\lambda_l'^2 b_l f_l^{11}(t) \exp\{im_{1l}(x-y)\}}{\lambda_l'^2},$$

$$R_l' = \frac{1}{\lambda_l'^2} \left\{ \lambda_l^2 a_l f_l^{11}(t) \exp\{im_{1l}(x-y)\} + \left(\lambda_l^2 - \lambda_l'^2\right) b_l f_l^{12}(t) \exp\{im_{1l}'(x-y)\} \right\},$$

$$M_l = \frac{1}{\lambda_l'^2} \left\{ -\left(\lambda_l^2 - \lambda_l'^2\right) a_l f_l^{11}(t) \exp\{im_{1l}(x-y)\} - \lambda_l^2 b_l f_l^{12}(t) \exp\{im_{1l}'(x-y)\} \right\},$$

$$M_l' = \frac{\lambda_l'^2 a_l f_l^{12}(t) \exp\{im_{1l}'(x-y)\}}{\lambda_l'^2}, \qquad L_l = -\frac{\lambda_l'^2 b_l f_l^{21}(t) \exp\{im_{2l}(x-y)\}}{\lambda_l'^2},$$

$$L_l' = \frac{1}{\lambda_l'^2} \left\{ \lambda_l^2 a_l f_l^{21}(t) \exp\{im_{2l}(x-y)\} + \left(\lambda_l^2 - \lambda_l'^2\right) b_l f_l^{22}(t) \exp\{im_{2l}'(x-y)\} \right\},$$

$$N_l = \frac{1}{\lambda_l'^2} \left\{ -\left(\lambda_l^2 - \lambda_l'^2\right) a_l f_l^{21}(t) \exp\{im_{2l}(x-y)\} - \lambda_l^2 b_l f_l^{22}(t) \exp\{im_{2l}'(x-y)\} \right\},$$

$$N_l' = \frac{\lambda_l'^2 a_l f_l^{22}(t) \exp\{im_{2l}(x-y)\}}{\lambda_l'^2}.$$

The elements of the P_l matrix are:

$$P_l^{11} = -\frac{F_l}{\sigma_l} \frac{\Phi_{l-1}^{22}(\lambda_l)}{a_l \Phi_{l-1}^{22}(\lambda_l) - b_l \Phi_{l-1}^{21}(\lambda_l)}, \qquad P_l^{12} = \frac{F_l}{\sigma_l} \frac{\Phi_{l-1}^{21}(\lambda_l)}{a_l \Phi_{l-1}^{22}(\lambda_l) - b_l \Phi_{l-1}^{21}(\lambda_l)},$$

$$P_l^{21} = -\frac{F_l'}{\sigma_l} \frac{\Phi_{l-1}^{22}(\lambda_l)}{a_l \Phi_{l-1}^{22}(\lambda_l) - b_l \Phi_{l-1}^{21}(\lambda_l)}, \qquad P_l^{22} = \frac{F_l'}{\sigma_l} \frac{\Phi_{l-1}^{21}(\lambda_l)}{a_l \Phi_{l-1}^{22}(\lambda_l) - b_l \Phi_{l-1}^{21}(\lambda_l)},$$

with

$$F_l = a_l f_l^{11}(t) \exp\{im_{1l}(x-y)\} + b_l f_l^{12}(t) \exp\{im_{1l}'(x-y)\},$$

$$F_l' = a_l f_l^{21}(t) \exp\{im_{2l}(x-y)\} + b_l f_l^{22}(t) \exp\{im_{2l}'(x-y)\},$$

$$\sigma_l = \frac{2\lambda_l'^2}{\lambda_l \left(\lambda_l'^2 - \lambda_l^2\right)}.$$

Construction of the nonlinear fields: Once the form of the matrices P_l and Q_l are determined we can construct the matrix B_l, so that the Lax eigenfunction $\Phi_l(\lambda)$ for the next stage can be determined from that of the previous one via,

$$\Phi_l(\lambda) = B_l(\lambda)\Phi_{l-1}(\lambda).$$

These expressions are very complicated, so we just quote one of them to display their structure. For example,

$$
\begin{aligned}
\Phi_l^{11}(\lambda) &= N_l/D_l, \\
D_l &= a_l \Phi_{l-1}^{21}(\lambda_l) - b_l \Phi_{l-1}^{22}(\lambda_l), \\
N_l &= R_l \Phi_{l-1}^{21}(\lambda_l)\Phi_{l-1}^{11}(\lambda) + \{R_l' - f_l(\lambda)F_l\}\Phi_{l-1}^{22}(\lambda_l)\Phi_{l-1}^{11}(\lambda) \\
&\quad + \{M_l + f_l(\lambda)F_l\}\Phi_{l-1}^{21}(\lambda_l)\Phi_{l-1}^{21}(\lambda) + M_l'\Phi_{l-1}^{22}(\lambda_l)\Phi_{l-1}^{21}(\lambda),
\end{aligned}
\tag{8}
$$

with similar expression for other elements Φ^{12}, Φ^{21} and Φ^{22}.

Now, for the determination of nonlinear fields, consider

$$
U_n' Q_n = M_n Q_n + Q_n U_{n-1}',
$$

where

$$
M = \begin{pmatrix} \partial_x & 0 \\ 0 & \partial_y \end{pmatrix}, \qquad
U_n' = \begin{pmatrix} -i\alpha & -\dfrac{q_n}{2} \\ -\dfrac{r_n}{2} & i\beta \end{pmatrix}, \qquad
U_{n-1}' = \begin{pmatrix} -i\alpha & -\dfrac{q_{n-1}}{2} \\ -\dfrac{r_{n-1}}{2} & i\beta \end{pmatrix}.
$$

In the expression for U_n' and U_{n-1}' we take $\alpha = \beta = 0$, which at once yields

$$
q_n = -\frac{2Q_{nx}^{12}}{Q_n^{22}} + \frac{Q_n^{11}}{Q_n^{22}} q_{n-1}.
\tag{9}
$$

This is nothing but a simple recursion relation. Similarly,

$$
r_n = -\frac{2Q_{ny}^{21}}{Q_n^{11}} + \frac{Q_n^{22}}{Q_n^{11}} r_{n-1}.
\tag{10}
$$

Explicitly are can write,

$$
n = 1, \quad q_1 = -\frac{2Q_{1x}^{12}}{Q_1^{22}} + \frac{Q_1^{11}}{Q_1^{22}} q_0, \quad r_1 = -\frac{2Q_{1y}^{21}}{Q_1^{11}} + \frac{Q_1^{22}}{Q_1^{11}} r_0,
$$

$$
n = 2, \quad q_2 = -\frac{2Q_{2x}^{12}}{Q_2^{22}} - \frac{2Q_2^{11}Q_{1x}^{12}}{Q_2^{22}Q_1^{22}} + \frac{Q_2^{11}Q_1^{11}}{Q_2^{22}Q_1^{22}} q_0,
$$

$$
r_2 = -\frac{2Q_{2y}^{21}}{Q_2^{11}} - \frac{2Q_2^{22}Q_{1y}^{21}}{Q_2^{11}Q_1^{11}} + \frac{Q_2^{22}Q_1^{22}}{Q_2^{11}Q_1^{11}} r_0.
$$

So far we have considered $f_l^{11}(t)$, $f_l^{12}(t)$, $f_l^{21}(t)$, $f_l^{22}(t)$ to be functions of time or constants; m_{1l}, m_{1l}', m_{2l}, m_{2l}' to be arbitrary constants; a_l, b_l to be arbitrary constants for all l. Now assume that $f_l^{12}(t) = f_l^{21}(t) = 0$ and $b_l = 0$ for all l values. So that $R_l = M_l' = L_l = L_l' = N_l = F_l' = 0$ for all l. In this case the form of $B_l(\lambda)$ turns out to be:

$$
B_l(\lambda) = \begin{pmatrix} \frac{\lambda_l^2}{\lambda_l'^2}\left(1 - \frac{\lambda_l'^2 - \lambda_l^2}{\lambda^2 - \lambda_l^2}\right)f_l^{11}(t)e^{im_{1l}(x-y)} & \left(1 + \frac{\lambda_l^2}{\lambda^2 - \lambda_l^2}\right)\left(1 - \frac{\lambda_l^2}{\lambda_l'^2}\right)f_l^{11}(t)e^{im_{1l}(x-y)}\theta_{l-1} \\ 0 & f_l^{22}(t)e^{im_{2l}(x-y)} \end{pmatrix},
$$

where $\theta_{l-1} = \dfrac{\Phi_{l-1}^{21}(\lambda_l)}{\Phi_{l-1}^{22}(\lambda_l)}$.

To write the formulae for the n-soliton solution in a compact form, we note that

$$Q_n^{11}Q_{n-1}^{11}Q_{n-2}^{11}\ldots Q_R^{11} = A_{n-R+1}^n F_{n-R+1}^n(t)\exp\left\{i\sum_{j=1}^{n-R+1} m_{1,n-j+1}(x-y)\right\},$$

where A_{n-R+1}^n, $F_{n-R+1}^n(t)$ stands for

$$A_{n-R+1} = \prod_{l=1}^{n-R+1} \frac{\lambda_{n-l+1}^2}{\lambda_{n-l+1}'^2}, \qquad F_{n-R+1}^n(t) = \prod_{l=1}^{n-R+1} f_{n-l+1}^{11}(t).$$

Similar expressions can be written for Q_n^{ij} and its products. Using these, we at once obtain:

$$q_n = \frac{-2\overline{m}_n Q_n^{12}}{G_1^n(t)\exp\{im_{2,n}'(x-y)\}} + \frac{A_n^n F_n^n(t)\exp\left\{i\sum_{j=1}^{n} m_{1,n-j+1}(x-y)\right\}}{G_n^n(t)\exp\left\{i\sum_{j=1}^{n} m_{2,n-j+1}(x-y)\right\}}$$

$$-2\sum_{K=1}^{n-1} \frac{A_K^n F_K^n(t)\exp\left\{i\sum_{j=1}^{K} m_{1(n-j+1)}(x-y)\right\}}{G_{K+1}^n(t)\exp\left\{i\sum_{j=1}^{K+1} m_{2(n-j+1)}(x-y)\right\}}\overline{m}_{n-K}Q_{n-K}^{12},$$

where we have set

$$\overline{m}_0 = a(m_0 - n_0), \quad G_K^n(t) = \prod_{l=1}^{K} f_{n-l+1}^{22}(t), \quad \overline{m}_n' = -im_{1n} + \overline{m}_0',$$

$$\overline{m}_0' = b(1/m_0 - 1/n_0), \quad F_k^n = \prod_{l=1}^{k} f_{n-l+1}^{11}(t), \quad T_n = \frac{m_0}{n_0}\left(1 - \frac{\lambda_n^2}{\lambda_n'^2}\right),$$

$$Q_n^{12} = T_n f_n^{11}(t)\exp(im_{1n}x)\exp(\overline{m}_{0x} + \overline{m}_n'y + \delta t).$$

Here δ is a complex constant.

Discussions: In the above analysis we have demonstrated how the pole type ansatz of Zakharov *et al* [3] can be used to generate a compact formulae for the N-fold Bäcklund transformation in the case of the Davey-Stewartson equation. The study yields two main results exhibited in equations (8) and (9). While the equation gives a recursive procedure for the determination of the Lax eigenfunction (Φ_l corresponds to the l-soliton state) equation (9) and (10) gives the corresponding recursion relation for the nonlinear fields. We have actually checked that for $n = 1$ one obtains the one soliton solution well known in the literature.

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